

B.Sc. Semester - 1 (CBCS) Examination
Oct/Nov-2025 (New Course)
MATHEMATICS (CORE)

Time: 2:30 Hours

Marks:70

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Attempt any two out of three (10)

- 1) State and prove Rolle's theorem
- 2) Show that $\frac{x}{1+x} < \log_e(1+x) < x; x > 0$.
- 3) Assuming the validity of expansion; expand $\log_e(1 + \sin x)$ up to first five terms; where $-\frac{\pi}{2} < x < \frac{3\pi}{2}$.

Que.1(B) Answer any one out of two (04)

- 1) State and prove Cauchy's mean value theorem.
- 2) Expand $\sin(x)$ in ascending powers of $(x - \frac{\pi}{2})$.

Que.2(A) Answer any two out of three (10)

- 1) Evaluate: $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{6/x^2}$.
- 2) Prove that the necessary and sufficient conditions for the differential equation $M(x, y)dx + N(x, y)dy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.
- 3) Evaluate: $\lim_{x \rightarrow \frac{\pi}{2}} (\cot^2 x - \frac{1}{x^2})$.

Que.2(B) Attempt any one out of two (04)

- 1) Evaluate: $\lim_{x \rightarrow 0} \frac{\sin x - x + (x^3/6)}{x^5}$.
- 2) Solve: $x \frac{dy}{dx} + y = x^3 y^6$.

Que.3(A) Attempt any two out of three (10)

- 1) Solve: $y = 2px + y^2 p^3$.
- 2) Solve: $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$.
- 3) Discuss the complementary function of $f(D)y = X$, when all the roots of auxiliary equation are real and distinct.

Que.3(B) Attempt any one out of two (04)

- 1) Solve: $y = px + p - p^2$.
- 2) Solve: $\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 13y = 0$.

Que.4(A) Attempt any two out of three (10)

- 1) In usual notation prove that $\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}; f(a) \neq 0$.
- 2) Solve: $(D^2 + 3D + 2)y = e^{2x} \sin(x)$.
- 3) Solve: $\frac{d^2 y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$.

Que.4(B) Attempt any one out of two (04)

- 1) In usual notation, prove that $\frac{1}{D-a} X(x) = e^{ax} \int X(x) e^{-ax} dx$.
- 2) Find the particular integral of the differential equation $\frac{d^2 y}{dx^2} + \frac{dy}{dx} + y = \sin(2x)$.

Que.5(A) Attempt any two out of three

(10)

- 1) Solve: $x^2 \frac{d^2 y}{dx^2} + y = 3x^2$.
- 2) Solve: $x^2 \frac{d^2 y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$.
- 3) Solve: $x^3 \frac{d^3 y}{dx^3} - x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} - 2y = x^3 + 3x$.

Que.5(B) Answer any one out of two

(04)

- 1) Find the complementary function of $x^4 \frac{d^3 y}{dx^3} + 2x^3 \frac{d^2 y}{dx^2} - x^2 \frac{dy}{dx} + xy = 1$.
- 2) In usual notation prove that $\frac{1}{f(\theta)} x^m = \frac{x^{m \log x}}{\varphi(m)}$; where $f(\theta) = (\theta - m) \varphi(\theta)$ and $\varphi(m) \neq 0$.
